Revealing traces of deterministic chaos in the accreting black holes

Petra Suková*, M. Grzedzielski and A. Janiuk

Center for Theoretical Physics, Polish Academy of Sciences, Warsaw; * psukova@cft.edu.pl

Introduction

The high energy radiation emitted by black hole X-ray binaries originates in an accretion disk. Most of the sources undergo fast and complicated variability patterns on different timescales. The variations that are purely stochastic in their nature, are expected since the viscosity of the accretion disk is connected with its turbulent behaviour induced by magnetic instabilities. The variability of the disk that reflects its global evolution governed by the nonlinear differential equations of hydrodynamics may not be only purely stochastic. Instead, if the global conditions in the accretion flow are such that the system finds itself in an unstable configuration, the large amplitude fluctuations around the fixed point will be induced. The observed behaviour of the disk will then be characterized by the deterministic chaos. The recent hydrodynamical simulations of the global accretion disk evolution confirm that the quasi-periodic flare-like events observed in couple sources are in a good quantitative agreement with the radiation pressure instability model of the disc coupled with strong outflows in form of a wind. At least 8 of the known BH X-ray binaries should have their Eddington accretion rates large enough for the radiation pressure instability to develop. In the current work, we aim to tackle the problem of stochastic versus deterministic nature of the BH accretion disk variability from the analytic and observational point of view.

Method

We use the capabilities of the recurrence analysis, which is a powerful tool for studying the time series and is known to work in broad range of applications (Marwan et al., 2007).
We first pose the "null hypothesis" about the measured time series, that the data are product of linearly autocorrelated gaussian noise. Then according to this null hypothesis we produce the set of surrogate time series sharing the spectrum and the value distribution with the original time series. This is achieved by an iterative algorithm called Iterative Amplitude Adjusted Fourier Transform Algorithm (IAAFT). We construct surrogates using the publicly available software package TISEAN (Schreiber & Schmitz, 2000).
The second order Rényi entropy K_2, which is a measure of positive Lyapunov's exponents and indication of deterministic chaos, is estimated for the observation and its surrogates using the program `rp` described by Marwan et al. (2007) and the significance of the difference between the real and artificial data is measured.

Recurrence analysis

The basic object of the analysis is the recurrence matrix, which describes the times, when the trajectory returns close to itself (closer than certain threshold ϵ). The recurrence matrix (usually visualised as recurrence plot – RP) is defined as follows:

$$\mathbf{R}_{i,j}(\epsilon) = \Theta(\epsilon - \parallel \vec{y}_i - \vec{y}_j \parallel), \qquad i,j = 1,...,N,$$

where $\vec{y}_i = \vec{y}(t_i)$ are (N) points of the reconstructed phase trajectory and Θ is the Heaviside step function.
The quantification of such visual information is contained in the histogram of diagonal lines of a certain prescribed length l,

$$P(\epsilon, l) = \sum_{i,j=1}^{N} (1 - R_{i-1,j-1}(\epsilon))(1 - R_{i+l,j+l}(\epsilon)) \prod_{k=0}^{l-1} R_{i+k,j+k}(\epsilon).$$

Because Rényi's entropy K_2 is related with the cumulative histogram of diagonal lines $p_c(\epsilon, l)$, describing the probability of finding a line of minimal length l in the RP, by the relation

$$p_c(\epsilon, l) \sim \epsilon^{D_2} e^{-l\Delta t K_2},$$

we can estimate the value of K_2 as the slope of the logarithm of the cumulative histogram versus l for constant ϵ.

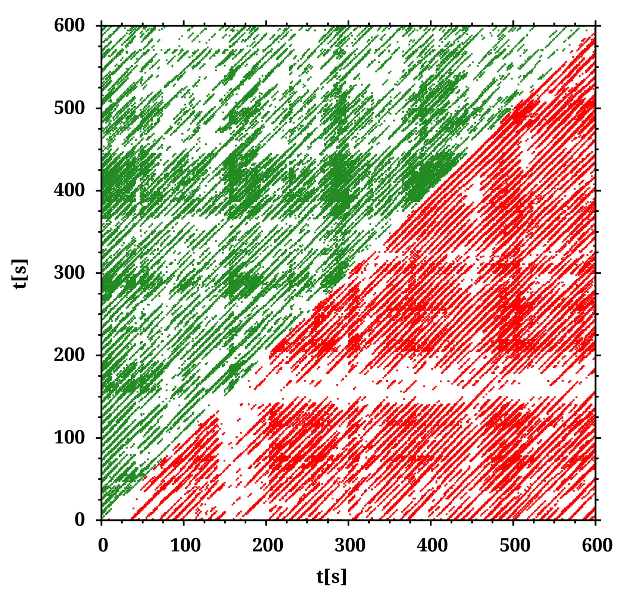

Fig 2 The example of RP of observation a) in Fig. 1 (red color, right lower corner) and one of its surrogates (green color, left upper corner). The real observation contains higher number of longer diagonal lines with more regular spacing. The noise causes the break of lines.
The plot was computed for $\epsilon = 4.4, \Delta t = 1.75$s, $m = 20$.

Phase space reconstruction

Measured data do not provide the phase space trajectory, it has to be reconstructed from the observed time series with the time delay technique. The resulting phase space vector is given as

$$\vec{y}(t) = \{x(t), x(t+\Delta t), x(t+2\Delta t), \ldots, x(t+(m-1)\Delta t)\},$$

where $x(t)$ is the time series, Δt is the embedding delay and m is the embedding dimension.

X-ray observations

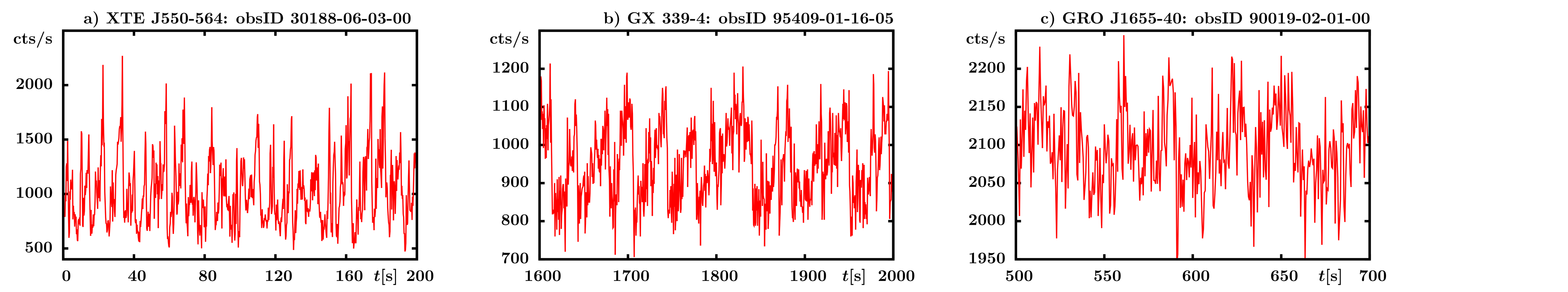

Fig 1 The examples of studied lightcurves for three different sources. The source names and obsID are indicated in the plots.

We analyzed the online data from the RXTE satellite for 6 different microquasars (IGR J17091-3624, GRS 1915+105 GRO J1655-40, GX 339-4, XTE J1550-564, XTE J1650-500). We extracted the time series using `Heasoft 6.16` high energy astrophysics software package. The resulting count rate is normalized to number of PCUs, which were used for the data extraction. Figure 1 show three examples of studied lightcurves from three different sources. Lightcurves a) and c) were extracted from Standard 1 data mode with the full energy range 2-60 keV, the lightcurve b) was extracted using generic event data mode from channels 5-25 ($\sim$ 2-10 keV).

Significance of the non-linear dynamics

The first hint about the non-linear dynamics hidden in the data is that the longest diagonal line $L_{\max}$ in the RP of the observation is longer than those from RP of the surrogates. However, this is not a definite answer, because K_2 is related with the slope of the cumulative histogram, rather than with the end of the histogram. The example of dependence of $L_{\max}$ on the recurrence threshold ϵ with $m = 16, \Delta t = 1.75$s for the observation a) (red line) and its surrogates (grey lines) can be seen in Fig. 3a).
In Fig. 3b) we show the cumulative histogram of diagonal lines and the slope of its part used for the estimation of K_2^{obs} (red line).
We define the significance of the obtained result so that it expresses how much the value K_2^{obs} differs from the mean value $\bar{K}_2^{\mathrm{surr}}$ measured in the units of the standard deviation of the set $\{K_2^{\mathrm{surr}}\}_{i=1}^{100}$ in the logarithmic scale $\sigma_{Q^{\mathrm{surr}}(\epsilon)}$:

$$\mathcal{S}(\epsilon) = \frac{N_{\mathrm{sl}}}{N^{\mathrm{surr}}} \mathcal{S}_{\mathrm{sl}} - \mathrm{sign}(Q^{\mathrm{obs}}(\epsilon) - \bar{Q}^{\mathrm{surr}}(\epsilon)) \frac{N_{\mathcal{S}_K}}{N^{\mathrm{surr}}} \mathcal{S}_{K_2}(\epsilon),$$

where N_{sl} is the number of surrogates, which have only short diagonal lines, and N^{surr} is the total number of surrogates, Q^{obs} and Q^{surr} are the natural logarithms of K_2 entropy for the observed and surrogate data, respectively, $\mathcal{S}_{\mathrm{sl}} = 3$ and $\mathcal{S}_{K_2}$ is the significance computed only from the surrogates, which have enough long lines according to the relation

$$\mathcal{S}_{K_2}(\epsilon) = \frac{|Q^{\mathrm{obs}}(\epsilon) - \bar{Q}^{\mathrm{surr}}(\epsilon)|}{\sigma_{Q^{\mathrm{surr}}(\epsilon)}}.$$

For further details, the reader is referred to Suková et al. (2015).
The significance depends on the parameters of the recurrence analysis (ϵ, m, Δt). The dependence of $\mathcal{S}$ on ϵ for several different values of m is in Fig. 3c). The values depend on ϵ, however no apparent trend can be seen.
Our final quantity is an averaged significance $\bar{\mathcal{S}}$ over interval of ϵ chosen so that recurrence rate RR $\in$ (1%, 25%) (RR is the ratio of the number of recurrence points to all points of recurrence matrix). The resulting $\bar{\mathcal{S}}$ for different m are in Fig. 3d). In agreement with Thiel et al. (2004), the dependence on m is quite weak, which supports our approach, in which we always use only one set of $[\Delta t, m]$ for each computation. Values of these parameters are estimated using the procedures `mutual` and `false_nearest` from `TISEAN`.

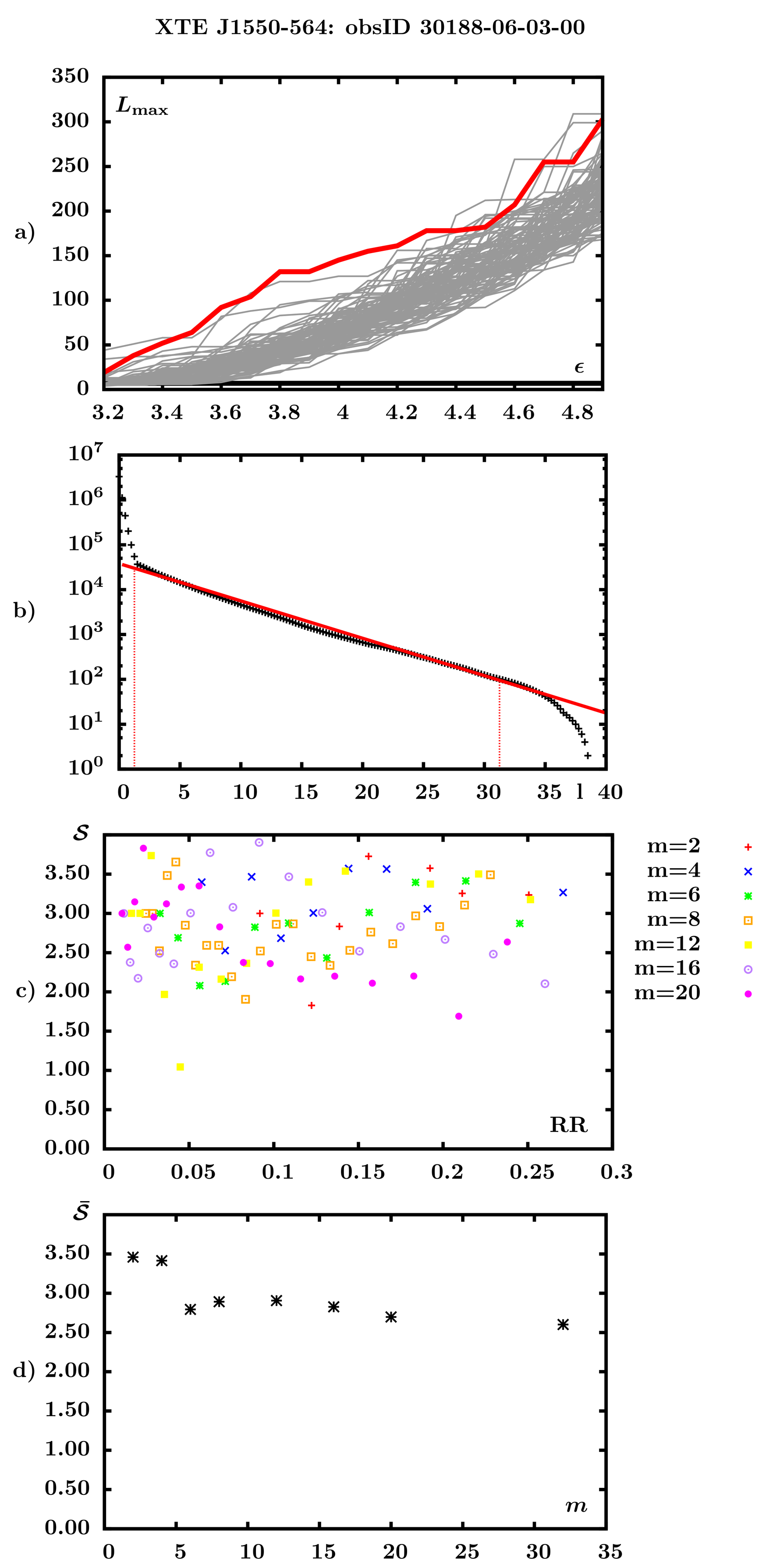

Fig. 3 Our analysis of XTE J1550-564.

Discussion

We applied the recurrence analysis on observations of six black hole X-ray binaries observed by RXTE satellite. We developed a method for distinguishing between stochastic, non-stochastic linear and non-linear processes using the comparison of the quantification of recurrence plots with the surrogate data. We tested our method on the sample of observations of the microquasar IGR J17091-3624. Significant results for the "heartbeat" variable ρ state were obtained. We examined several observations of the other five microquasars. Aside from the well-studied binary GRS 1915+105, we found significant traces of non-linear dynamics also in three other sources (GX 339-4, XTE J1550-564 and GRO J1655-40). The fact, that we have found non-linear dynamics hidden in the light curves of almost all the studied sources means, that in genereal the evolution of the accretion disc and corona is important for the outgoing radiation. The non-linear behaviour of the lightcurve during some of the observations gives the evidence, that the accretion flow in the binaries is governed by low number of non-linear equations. Possible explanation is that the accretion disc is in the state prone to the thermal-viscous instability and is undergoing the induced limit cycle oscillations.
Our studies show, that the non-linear variability due to chaotic processes appears in the disk-dominated soft state and the intermediate states of the microquasars. It is planned to carry the study of non-linear dynamics for microquasars together with a study of the whole outburst evolution and link it with the spectral state transitions and hardness-intensity changes of a given source. Also, other characteristics, such as the presence and properties of winds launched during some states from their accretion discs, should be taken into account in our future work.

References & Acknowledgement

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